11th International Conference on Higher Education Advances (HEAd'25)
Universitat Politècnica de València, València, 2025
DOI: https://doi.org/10.4995/HEAd25.2025.20116

Exploring the integration of Mathematics and Music within the STEAM Framework: Insights from the Medieval Quadrivium

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How to cite: De Castris, R.; Tortoriello, F. S.; Veronesi, I. (2025). Exploring the integration of Mathematics and Music within the STEAM Framework: Insights from the Medieval Quadrivium. In: 11th International Conference on Higher Education Advances (HEAd'25). Valencia, 17–20 June 2025. https://doi.org/10.4995/HEAd25.2025.20116

Abstract
This paper explores the relationship between the medieval Quadrivium and contemporary STEM/STEAM methodologies, providing a valuable insight into educational practices. The Quadrivium conceived knowledge as an integrated entity, a concept that is reflected in contemporary STEM/STEAM model, which aims to overcome compartmentalized teaching by promoting critical thinking and creativity. Both approaches favour holistic learning, in which knowledge is not fragmented but part of a unified, interdisciplinary system. An experimental path is proposed in line with the activities of the Mathematical High School Project at the University of Salerno (Italy): an activity exploring the classical isoperimetric problem through mathematics, science, engineering and music. The study of mathematics becomes more concrete, engaging and meaningful, preparing students for an increasingly dynamic and multidisciplinary world.

Keywords: *Interdisciplinarity; STEAM; Mathematics; Music; Mathematical High School Project; Technology enhanced learning.*

1. Introduction

The connection between the Quadrivium model and the modern STEM (Science, Technology, Engineering and Mathematics) or STEAM (with the addition of Art) approach is very significant and offers a perspective that enriches the way of teaching by offering an integrated view of knowledge. The Quadrivium, literally "crossing of four roads", is an educational model first developed in the Pythagorean school. Around 500 B.C. Pythagoras founded a community in Croton based on equality in both moral and material terms. It was the first to divide education into two parts: the Trivium and the Quadrivium. The Trivium focused on language and expression—Grammar, Logic, and Rhetoric—key for public life and philosophy. The Quadrivium explored number and its role in understanding the universe, with four disciplines: Arithmetic (numbers), Geometry (space), Harmony/Music (time), Astronomy (motion). The importance of the Quadrivium didn't lay only in the acquisition of practical skills, but above all in the search for a higher order that would harmonize human knowledge with universal principles. Many philosophers and intellectuals contributed to the development of the Quadrivium, including Plato, Aristotle and Euclid. Throughout the Middle Ages and subsequent periods, scholars such as Dante, Kepler, Al-Kwarizmi investigated the connections among mathematics, music, astronomy and philosophy.

In Plato's Republic (Book VII), music guides the soul toward truth by teaching harmony and proportion. Geometry reveals eternal truths, encouraging philosophical thought. Astronomy should be studied like geometry—focused on problem-solving rather than observation—to strengthen the soul's intellect. Ultimately, these studies lead to the ideals of the True, the Beautiful, and the Good. Through the harmony of numbers and proportions, individuals can gain insights into the mechanisms of the cosmos and enhance their spiritual awareness (Ferguson, 2008), (De Maria, 2018). In Phaedo, Plato claims the soul is immortal and recalls truths known before birth—learning is thus remembering, or anamnesis. Education, then, is an inner search for true knowledge, not just information transfer. This principle reinforces the idea that education should not be a mere transfer of information, but an inner journey in search of authentic knowledge. The Quadrivium supported this by placing math at the heart of learning and viewing knowledge as a unified whole—a concept echoed today in STEM/STEAM, which blends disciplines to foster critical thinking and creativity (Sternberg, 1988).

STEM/STEAM is an acronym that represents a multidisciplinary educational framework emphasizing the integration of the disciplines and enabling students to recognize the relationships among these disciplines and their practical applications in addressing real-world challenges. It extends beyond merely acquiring new knowledge, but also of developing skills such as analytical reasoning (Dewey, 1938), (Bloom, 1956), divergent thinking (Guilford, 1967), experiential learning, learning by doing (Lewin, 1951), skills that are fundamental to face a future in which technology and science will play a leading role. The presence of art in enhances the framework by transcending mere technical and scientific reasoning. Art, in fact, is about the ability to think creatively (Rochon, 2024), to look at problems from different angles, making solutions more fascinating and often more efficient. In "The Seven Knowledges Necessary for Future Education" (Morin, 2001), Morin emphasizes the importance of replacing thinking that separates and reduces with thinking that distinguishes and connects. This shift does not entail abandoning knowledge of parts for knowledge of totality, nor replacing analysis with synthesis; rather, it requires an integration of both approaches.

Imagine projecting the Quadrivium disciplines into STEM/STEAM disciplines. From arithmetic to mathematics: studying numbers and their properties develops the basis for algebra, cryptography and data analysis, all key skills for programming and AI; from geometry to engineering: proportions and geometric shapes (geometry) are fundamental in structural engineering, design and digital graphics (Elam, 2001); from music to technology: music is not only art, but also music software, sound physics, wave theory (Camurri, 1990), (Boschi, 1991), (Ozzola, 2018); from cosmology to science: the study of celestial movements and cosmic proportions is the basis of astronomy and space exploration (James, 1993).

Certainly, Quadrivium has a more theoretical and philosophical focus, while STEM is more application and technology-oriented, but both promote a holistic approach to learning, in which knowledge is not fragmented but part of a unified, interdisciplinary system. The importance of this approach in education is based precisely on the intersection of theory and practice and on interdisciplinarity, which helps students to grasp the connections between different subjects, provides tangible experiences, increases motivation and enables problem solving, an essential skill for modern careers that increasingly demand mixed skills.

2. The Mathematical High School Project

The educational initiative presented in this document is strategically crafted as a component of the Mathematical High School Project (MHS, in Italian “Liceo Matematico”) and is intended for implementation in 12th and 13th-grade classrooms. The MHS project involves a collaboration among 30 universities and more than 160 educational institutions across Italy.

This forward-thinking educational program seeks to reintegrate the interdisciplinary essence of mathematics into the Italian school curriculum. It is designed through hands-on laboratory experiences and cross-disciplinary courses led by skilled educators and university researchers specializing in pedagogy and educational development, conducted during extracurricular hours.

The objective of this initiative is to enhance the connections between scientific and humanistic disciplines, fostering a variety of interdisciplinary approaches. By leveraging emerging technologies, the project seeks to explore and reinterpret subjects such as history, art and economics through a mathematical perspective (Bologna et al., 2020), (Rogora &Tortoriello, 2021), (Tortoriello & Veronesi, 2021), (Bimonte et al., 2023), (Musmarra et al., 2023). Within the framework of this initiative, music serves as a significant unifying cultural force, harmonizing with mathematics to enhance cognitive development. The objective of this project is to develop a prospective interdisciplinary educational framework, initiating with the interplay between mathematics and music.

The methodological framework underpinning the activities within the MHS is founded on constructivist principles (Vygotsky, 1934). Constructivism is a theoretical framework that highlights the proactive engagement of learners in shaping their own comprehension of the world around them. According to this perspective, knowledge is not simply absorbed; instead, it is actively developed through social interactions and influenced by one's surroundings. The constructivist framework offers a versatile and inclusive methodology that facilitates the examination of various perspectives and the recognition of fundamental patterns and themes. In conclusion, employing a constructivist approach within this laboratory fosters a thorough comprehension of interdisciplinary subjects.

3. The Interdisciplinary Laboratory

3.1. Maths and Music with technology

Software plays a fundamental role in modern education, as it offers tools that enrich the learning experience, making it more dynamic, interactive and personalized. Thanks to educational software, applications and online platforms, teachers can integrate content by presenting it in an engaging way, stimulating students' curiosity, and facilitating the understanding of complex concepts. Moreover, they facilitate adaptive teaching (Sweller, 1988), capable of responding to students' individual rhythms and needs, promoting a more inclusive approach.

GeoGebra has long been an essential tool in mathematics education, as it allows mathematical concepts to be visualized and manipulated in a concrete way. Teachers can design hands-on activities that facilitate understanding of the relationships between different areas of mathematics and encourage the development of critical thinking. Interactivity and the ability to explore without fear of making mistakes make teaching more engaging and less abstract. GeoGebra also encourages collaboration between students, who can solve problems together, improving their mathematical and digital skills and offers a practical example of the multimedia learning theory (Mayer, 2001).

Music software also enhances learning, making it more interactive, practical and accessible. Muse Score, Finale, Sibelius are examples of software that are extremely useful both for teaching music theory and writing, and for developing listening and composition skills. Teachers can create spares that students can edit in real-time, and the sound playback feature helps connect the writing of notes with their actual sound.

3.2. The development of the learning path

In the proposed experimental activity, the software GeoGebra has been introduced as a semiotic mediator of mathematical meanings. In particular, the isoperimetric problem is a classic optimization problem: "Among all plane figures that have a specified perimeter, which configuration yields the maximum area?". The quest to identify the figure with the largest area, while maintaining a constant perimeter, offers an ideal interdisciplinary approach integrates concepts from Quadrivium and STEM.

The objective was to design an educational activity to explore the link between mathematics, music, arithmetic, astronomy and the STEM disciplines. This initiative aims to link the geometric optimization of the isoperimetric problem to harmonic optimization in music, as well as to arithmetic principles, cosmic proportions and technological applications. Demonstrating how all these fields share structures and proportions, the laboratory invites to reflect on mathematics influences both in the sciences and arts. The parallelisms in art, music, geometry, cosmology and engineering, offer students an interesting and creative interdisciplinary vision.

The aim of the activity is to engage students in the analysis of the harmonic ratios that define musical intervals (e.g., 2:1 for the octave, 3:2 for the right fifth, 5:4 for the major third) (Corona, 2015). Furthermore, it aims to establish connections between these ratios and the geometric proportions of the isoperimetric problem, using mathematical, technological and practical tools. Additionally, it investigates the relationship between these ratios and the fundamental numerical proportions in arithmetic, the way in which they are highlighted in the organization of planetary orbits in astronomy and their applications in modern technology.

The prerequisites required are a basic knowledge of plane geometry (familiarity with the concepts of proportion and ratio), of dynamic geometry software (such as GeoGebra) and music software (such as Muse Score), of introductory notions of music theory (such as scale structure and intervals). A basic understanding of rational and irrational numbers in arithmetic and the structure of the solar system in astronomy is also useful.

The activity begins by introducing the classical isoperimetric problem and comparing the idea that the circle represents the maximum contained area for a given perimeter with the search for perfection in musical ratios and astronomical proportions, highlighting how all these areas seek to maximize a certain characteristic (area, harmony or cosmic order).

Students construct regular isoperimetric polygons with an increasing number of sides and compare their areas using the dynamic geometry software GeoGebra. They observe that as the number of sides increases, the area increases and recognize that the circle (polygon with an infinite number of sides) has a maximum area, as seen in the figure.

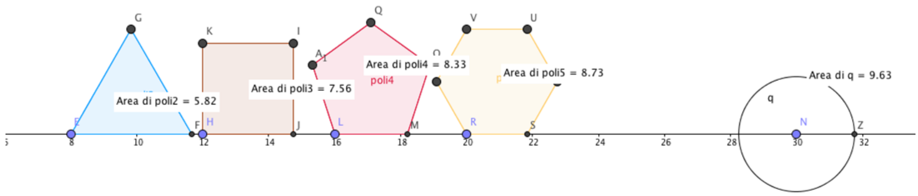

Figure 1 - Classical isoperimetric problem and polygon comparison

It is emphasized to the students that the graphical experimental verification does not constitute a mathematical demonstration, the heuristic approach to the problem is effective in fostering didactic reflection. Then, students use GeoGebra to represent a circle and divide it into arcs proportional to harmonic ratios.

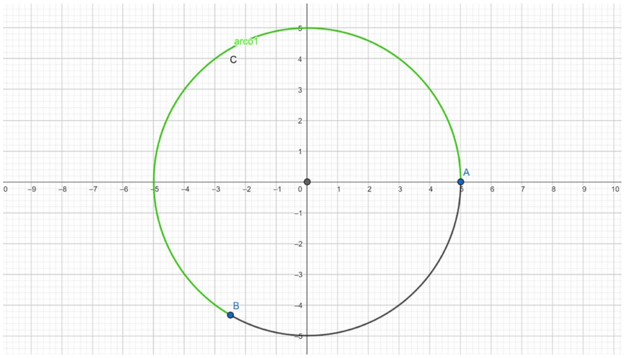

Figure 2 – Example of an arc of 2/3 of the length of the perimeter representing the perfect fifth

Following, students are asked to construct regular polygons inscribed in the circle, where each side represents a note.

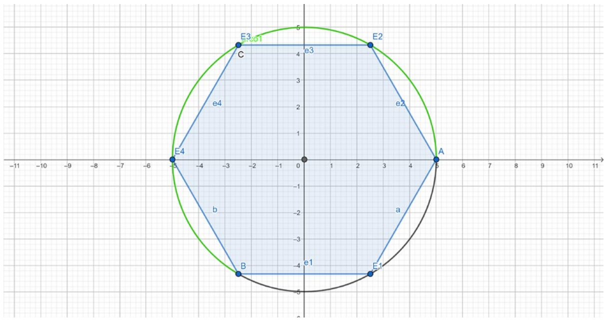

Figure 3 – The example of the regular hexagon (n=6)

Simultaneously, they examine the fundamental numerical series underlying these relationships in arithmetic and their correlation with the movements of astronomical objects and modern satellite navigation technologies, where the determination of optimal satellite trajectories and synchronization exploit advanced geometric concepts.

The activity proceeds with students calculating the ratios between the sides and verifying whether they correspond to known harmonic intervals. To gain a comprehensive understanding of the ratios and connections with musical intervals, software tools can be used to dynamically change the length of the arcs. Additionally, the analysis of proportions in planetary orbits is noteworthy, such as the Titius-Bode law, and how they reflect universal harmony.

Finally, with the Muse Score software, students can reproduce the frequencies corresponding to the harmonic ratios. This practice facilitates a deeper understanding of the auditory effects associated with mathematical proportions. Students may be encouraged to listen to the generated chords and verify how mathematical ratios influence the perception of harmony.

The activity encourages a thoughtful examination of the classical isoperimetric problem as an example of geometric optimization drawing a parallel to harmonic optimization in music, the numerical structure of arithmetic, the harmonic arrangement of the universe in astronomy and modern STEAM applications.

4. Conclusions

This paper advocates for a reinstatement of a cognitive approach reminiscent of the Quadrivium, which emphasizes the harmony between the mind, the senses, and the universe. It proposes an educational model centered on the significance of constructing enduring meanings that are collectively understood across various dimensions of knowledge.

The educational path presented in this article is part of a constructivist learning model that uses mathematics to foster integration between disciplines. The objective is to build upon the Quadrivium tradition while integrating contemporary STEM/STEAM methodologies to create a teaching experience in which students actively engage in their own learning process (Vygotsky, 1934).

The proposed laboratory activity is centered on the well-known isoperimetric problem enhanced through its integration with music and goes beyond a mere fusion of disciplines. It is rooted in the concept of "connection with a purpose." (Bauman, 2003). Unlike the superficial and temporary connections typical of liquid modernity, a connection with a purpose is oriented towards the construction of meaning and lasting relationships. In this laboratory, in fact, students do not limit themselves to relating mathematical and musical concepts, but build a stable bridge between languages, recognizing in the harmonic structure of music the same principles of proportion and order that govern geometry. The laboratory thus becomes a place of deep relationships: between disciplines, between logical thinking and aesthetic sensitivity, between students and teachers, between theory and practice. This didactic approach promotes knowledge that is not aimed at immediate consumption, but at critical elaboration and reflection, dialogue between intelligences, and the shared construction of meanings. It is not just a method, but a form of thinking that breaks down the barriers between subjects.

The Quadrivium-STEM/STEAM combination succeeds in creating an innovative learning environment that enhances both ancient knowledge and the possibilities offered by new technologies. In fact, through the combined use of software such as GeoGebra and Muse Score, students can gain insight into the fundamental role that mathematical proportions play in both geometric optimization and musical harmony. Additionally, they will learn how these concepts are interconnected with astronomy, particularly in the organization of planetary orbits and the advancements in modern navigation technology.

The objectives can be categorized into several domains of learning, which encompass the comprehension of mathematical concepts, the cultivation of interdisciplinary skills, the enhancement of technological proficiency, the attainment of critical and reflective thinking abilities, the development of problem-solving skills, and the encouragement of collaborative learning.

The classroom experimentation with students will begin in May 2025, and will be accompanied by a phase of data collection and analysis for further critical reflection. At the same time, the workshop will be proposed as a training course in refresher courses for teachers, with the aim of disseminating interdisciplinary teaching practices that respond to contemporary educational challenges.

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