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OPS4Math project -Optimization and Problem Solving for Teaching of Mathematics: teaching strategy, organization and objectives

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Abstract

Several initiatives have been implemented worldwide to foster student interest towards STEM disciplines. These initiatives are based on the awareness that mathematics is essential for scientific and technological advancement: it trains to reasoning and reflection, stimulates logical capabilities and intuition, improve investigation attitude. Most of them recognize also that mathematical problem solving represents an effective way to support teachers and students in their teaching and learning activities, respectively. In this context, this work is aimed at presenting OPS4Math (Optimization and Problem Solving for Teaching of Mathematics), a training project for Secondary School teachers, supported by Italian Ministry of University and Research. The driving idea, widely discussed by the scientific community, is to operate a reversal of the didactical perspective: starting from phenomena/problems to introduce concepts of data, variables, relationships and functions in an appealing way. We present project organization, structure and aims, to give useful hints for its replication.

Keywords: *Problem solving; optimization; teaching and learning strategy.*

1. Introduction and project context

Nowadays, the teaching of science, technology, engineering and mathematics, i.e., the so called STEM disciplines, represents a relevant topic of discussion given their boosting role for the economic growth of any developed and developing country. Roughly speaking, we can say that teaching STEM disciplines can be conceived as teaching skills and subjects in a way that resembles real life. Indeed, through STEM education, students learn facts and a method of thinking/reasoning that can be applied to solve real complex and multidimensional problems. Becoming a STEM professional requires may years of preparation to gain academic and job-specific knowledge through higher learning institutions and field training. Such knowledge defines the so called hard skills or technical skills. However, this knowledge is not sufficient if it is not integrated by the acquisition of the so called professional soft skills and competences, which are fundamental to succeed in a career path (e.g., communication skills, problem-solving, critical thinking, teamworking, and digital skills) (Lavi et al. 2021). Such skills, deriving from personal habits and personality, define the way one works on his own and with the others. As reported in Szabo et al. (2020), the importance of such skills has been widely discussed at several levels by OECD (Organization for Economic Co-operation and Development), and by EU (European Union). As further confirmation, in many job descriptions, employers often ask for a combination of hard and soft skill. Thus, without fear of being proven wrong, we can say that this combination represents a key enabling and competitive factors to face the STEM challenges of modern societies.

In this context, the need arises of accompanying and supporting the hard skill education at school and university level with the introduction of soft skills in classical STEM education curricula (Falloon et al. 2020). This target is shared but it is also recognized that teaching soft skills is much harder with respect to hard skills. As evidence of this, numerous studies highlight that there continues to be a skills mismatch between graduates and job market.

This work deals with problem solving, and more precisely, on mathematical problem solving, which is likely one of the most discussed soft skills in literature, given its central role in STEM professions because of its relationship with mathematics teaching and education. Moreover, we will focus only on the Italian situation. The literature is rich of problem-solving definitions differing in the number of actions to be performed. However, all of them, share the following concepts and phases defined by Pólya (1945) in his so-called heuristic method: problem and goal definition; problem solution and generation of the solutions; implementation of one or more solutions; evaluation of the results and feedback operations for continuous improvement. These concepts can be easily transferred to the mathematical problem solving. Indeed, it is widely recognized that learning mathematics concern reasoning not memorization, or in other words, it concerns the development of processes to reach a solution, rather than the application of a set of procedures. Thus, problem-solving can be considered as a mean to engage students on complex tasks with the aim of: developing the

strategic thinking; providing a deeper understanding of mathematical concepts; fostering the appreciation of the relevance and usefulness of mathematics.

This view of teaching and learning mathematics in relation with problem solving and its application in STEM disciplines has been largely discussed also in Italy, where national institutions recognize the importance of promoting STEM education and the need of enhancing cross-curricular integration of mathematics (Ceselli and Righini, 2017). In this context, several national actions have been put in place. Among the others, we cite here: the "Scientific Degrees Project" (Progetto Lauree Scientifiche, PLS), started in 2004, by which the government offers incentives to students who enrol in STEM curricula; the initiative "Alternanza Scuola-Lavoro (ASL)", started in 2015, which foresees secondary school students to spend a prescribed yearly number of hours in companies and other institutions rather than at school; finally, from 2018, the initiative "PCTO – Percorsi per le Competenze Trasversali e l'Orientamento", which foresees training paths of at least 210 hours in the last high school three years, aimed at the development of student personal and transversal skills. Moreover, other local actions have been made in these years, more focused on the problem-solving skills and its connection with operations research. For the sake of the brevity, we address the interested reader to the work by Raffaele and Gobbi (2021). However, most of these initiatives are more focused on "*teaching mathematics for problem-solving*" rather than "*teaching mathematics through problem solving*". Thus, they do not reflect directly in actions devoted to the insertion of problem solving in school curricula, thus confirming the mismatch between the mathematics education provided by school and university and the required problem-solving skills.

This work is aimed at presenting a project providing a contribution in filling this gap. It is devoted to the presentation "OPS4Math – Optimization and Problem Solving for Teaching of Mathematics", a training project for Secondary School teachers organized by the University "Federico II" of Naples, financed by Italian Ministry of University and Research (D.D. 1662 del 22.10.2020), and supported by Campania Regional Education Office. The project driving idea, resembling the "teaching mathematics through problem solving" approach, consists in operating a reversal of the didactic perspective in classical curricula: starting from phenomena/problems to introduce concepts of data, variables, relationships and functions in an appealing way, providing new materials for mathematic teachers of Secondary School. We present project organization, structure and aims, to give useful hints for its replication at national and international level.

The rest of the work is organized as follows: *Section 2* briefly discuss the mathematical problem-solving approach, its usage in Italy and its implementation in OPS4Math project; *Section 3* is devoted to the presentation of the organizing team background and describes the overall project; finally, *Section 4* reports conclusions and a discussion about project future outcomes and challenges.

2. Problem-solving and teaching of mathematics

Nowadays, mathematics education provides the student with the set of mathematical tools and techniques (e.g., axioms, theorems, proofs, formulas, etc.) required by the school curricula. In other words, we could also say that it focuses on the essentials ingredients of mathematical knowledge and understanding the student is expected to manage when solving and practicing on the solution of given exercises (Schoenfeld, 2016).

However, two situations often occur in mathematical learning in Italy. On one side, it is widely recognized that mathematics appears to the students as a difficult subject. On the other side, students apparently know everything they need to know, but they are not able to transfer their mathematical knowledge in other situations. Both situations reflect a poor knowledge of mathematics which appears today as a weakness condition in all the situations where a numerical approach has to be adopted to solve problems arising in real life complex systems.

Mathematicians have always understood that problem-solving, allowing the application of mathematics in everyday life, has a central role in mathematics education since without a problem there is no mathematics. At the same time, it is also clear that mathematics teaching cannot be reduced to just reality-based examples. In this context, the mathematical problem-solving teaching approach arises as a good comprise solution between the applicative and theoretical aspects of mathematics teaching. Its main aim is to engage the students in applying their knowledge of mathematical concepts to real life problems, integrating and connecting isolated pieces of mathematical knowledge, making connections between problem information and mathematical operations, patterns, and rules (Klang et al, 2021). Problem solving and mathematical problem solving have been widely treated in literature over the past fifty years, starting from the work of Pólya (1945). A review of the complete body of research is beyond the scope of this work. Thus, we limit ourselves to address the interested to the contributions by Schoenfeld (2016) and Stohlmann and Albarracín (2016).

In the following, we will just focus on two different usages of mathematical problem-solving with respect to teaching of mathematics. Two main strategies are discussed in literature (Charles, 2009, and Klerlein and Hervey, 2019): "*teaching mathematics for problem solving*" *(TM-for-PS)* and "*teaching mathematics through problem solving*", (*TM-through-PS*).

As discussed in Schoenfeld (2016), *TM-for-PS* implies the following five underlying usages of problem solving: as a justification for teaching mathematics; to provide specific motivation for subject topics; as recreation; as a means of developing new skills; as practice. Thus, problem solving mainly represents a mean to facilitate the mathematical teaching and practicing mathematical instruments. Instead, *TM-through-PS*, is based on the conviction that introducing new concepts and skills in problem-solving context inspires thinking and reasoning about mathematical embedded ideas. Thus, problem solving represents a mean to teach mathematics in a way that makes sense to students.

It is easy to understand that both strategies should play a relevant role in mathematics teaching and education. However, most of the initiatives, made at Secondary School level in Italy in the last 15 years, are mainly focused on *TM-for-PS*. 1. To the best of authors knowledge, no initiative in Italy, has been focused on TM-through-PS.

2.1. Problem Solving approach in OPS4Math

The previous discussion on mathematical problem solving and related teaching strategies has a twofold aim: one side, motivating the need of developing a teaching strategy which well balances *TM-for-PS* and *TM-through-PS*; on the other side, highlighting the lack of *TM-through-PS* initiatives in Italy. Concerning this lack of *TM-through-PS* with respect to *TM-for-PS* initiatives, the main reasons can be summarized as follows:

1. The conviction that *TM-for-PS* initiatives have a faster impact on the pursued objective of improving the results of Italian students at OCSE-PISA tests.
2. It is easier for teachers to include *TM-for-PS* subjects in the current mathematics curricula. Such inclusion is also coherent with the way schoolbooks are written.
3. The usage of *TM-through-PS* strategy requires a significant change in the way lessons have to be made. Thus, teachers require professional learning and training experiences to support this change.

OPS4Math project, "Optimization and Problem Solving for Teaching of Mathematics", is aimed at supporting the integration of *TM-for-PS* and *TM-through-PS*, providing a contribution in terms of teachers training and material drafting, coherently with the issues raised in Anderson (2005) and Akhter et al. (2015). More precisely, it provides a training activity for Secondary School teachers of Mathematics in Campania Region, to present the mathematics curricular subjects by a problem-solving approach.

In this scheme, a special attention will be devoted to the didactical strategy and therefore to the relation between deductive and inductive reasoning. The Italian (mathematical) tradition is historically linked to the deductive approach, preferring a didactical path which starts from definitions and hypotheses to reach a thesis and a rule, applicable, whereby necessary, to the explanation of a physical or natural phenomenon and to the solution of a real problem. This approach determined the development of a rigorous (and necessary, sometimes compulsory) formal language, which does not encourage the approaching to mathematics and even facilitates the removal (estrangement) from it. This happens increasingly in this historical phase which sees the transition from the "*learning for reading"* to the *"learning for hearing and vision*". In the second half of the last century the relationship of Mathematics with real problems strongly developed, in connection with Computer Science, Artificial Intelligence and Data Science. Consequently, the teaching of Mathematics is subjected to a transformation process, devoted to put in contact the student with its application. Thus, in this moment, the

inductive reasoning appears to be more suitable to push students towards mathematics, appreciating his capability to analyse phenomena, modelling and solving real problems.

OPS4Math project operates in this direction. The mathematical problem-solving approach, and more precisely *TM-through-PS* strategy, allows to operate what we call the "*reversal of the didactical perspective*". Starting from phenomena and problems allows to introduce the concepts of data, variables, analytical relationships and functions in an appealing way. Mathematical modelling can be used to define the mathematics contents in a way which is not abstract and conceptual but linked to the reality. Simple decisional problems can be conceived as samples of real complex problems in the fields of traffic, transportation, environment, health and all other economic sectors. Thus, identifying and building models and developing action plans are crucial matters of the problem-solving approach to acquire mathematical knowledge. In this context, great relevance will also be given to optimization problems, solution algorithms and computational thinking, which are key elements for STEM disciplines. Thus, the name of the project OPS4Math, recalls its 4 pillars: Problem Solving, Optimization, Algorithms, Didactical strategies.

3. OPS4Math project

OPS4Math project will be developed by the Operations Research Group of the Optimization and Problem-Solving Laboratory (OPSLab) of the University "Federico II" of Naples. Such project arises from the experience gained by the group in the last 15 years of collaborations with education institutions. More precisely, the group participated in the following initiative. The Education Department of Campania Region promoted two courses, "Logimat" and "Logimat2" (in the period 2008 – 2010), focused on the logical-mathematical learning. The scope was to train mathematics teachers of Secondary School, within a deal between Public Education Ministry and Campania Region. After this experience the Campania Office of Education Ministry conducted another initiative devoted to developing the problem-solving approach in teaching of Mathematics in Secondary School. The project "OCSE PISA 2015 - Objective 500", had the aim to increase proficiency of the fifteen years old students of Campania to reach the score 500 at the OCSE PISA test. The project was biennial, and 80 schools participated to the initiative. Later, "Science Center of Naples" promoted the "LogicaMente", a national project to support the improvement of scientific, logic and mathematical skills of the students. In the last six years, the University "Federico II" of Naples constituted the F2S group ("Federico II in School") to build a link between School and University, with the aim of preparing students to tackle the university experience.

OPS4Math project is aimed at "*making a system*" of previous experiences. Indeed, as done in "Logimat" and "Logimat2", a training activity for teachers is foreseen, and, as done in

"OCSE PISA 2015 - Objective 500", a validation with students in classroom will be made. The following subsections will provide the project organizational details.

3.1. Phases of the project

The project is organised in 4 phases.

Phase 1: Preparatory phase and activity planning

- Meeting with the Regional Education Office for selection of the involved schools and teachers
- Meetings with the directors of the selected school
- Preparation of the educational material, software and multimedia instruments

Phase 2: Training of the teachers

The training activity will be performed through lessons and seminars held by experts in the field of mathematics, statistics, physics and operations research. They will focus on:

- Data collection and analysis, time series and graphical representation
- Introduction of mathematical relationships and functions by the definition and solution of real problems
- Topics of the curricular programme presented with a problem-solving approach
- Modelling of real decisional problems and related algorithms
- Software for teaching of mathematics

The class will be composed by 25-30 teachers. At the end of the course, the selected teachers, individually or in team, with the supervision of an expert, will prepare a short seminar/working paper, where they present a curricular subject by a problem-solving approach. Such material will be one of the outputs of the project.

he course is structured in 15 meetings, so defined: opening and project presentation; training lessons of about 4 hours; final meetings for the presentation of the work made by the teachers. Seminars and lessons will be held in blended form, to prevent the covid-19 risk and to support the participation of teachers living far from the venue of the meetings.

Phase 3: Field testing of the didactical strategy

A field testing of the didactical strategy will be performed with the teachers and the students of a set of schools selected in cooperation with the Regional Education Office. The students will be invited to express their opinion about the proposed didactical strategy and their appreciation to promote their participation in the didactical trial. The field testing foresees fifteen 2-hour meetings in the selected schools. The trained teachers could become trainers of other teachers, so activating a mechanism of pyramidal dissemination aimed to reach a larger number of students.

Phase 4: Dissemination of the results

A final workshop will be organised to present the results of the project, to debate on the theme and to hand the attendance certificate to the teachers. The results of OPS4Math will be broadly diffused through the website of the project (*ops4math.dieti.unina.it*), together with the didactical material, video, multimedia, tests and links to the literature.

3.2. Objectives

The main objectives of the project can be summarized as follows:

- Showing teachers and students that an alternative way of teaching of mathematics is possible, by studying and analysing phenomena and solving real problems.
- Verifying in the classrooms the response and the reaction of the students to the proposed didactical approach.
- Stimulating the study and the practice of the mathematical modelling, algorithms and software, to foster the study and the learning of mathematics and, more generally, STEM disciplines.

3.3. Contributors

Two scientific associations, working in the education field at school and university levels, are involved in the project: AIRO (Italian Association of Operations Research), established from 1961, to which Operations Research teachers and researchers join, mostly from the University; Mathesis (Italian Society of Mathematical and Physical Science), established from 1895, to which Mathematics and Physics teachers join, mostly from the School.

Several experts coming from these associations will be involved in the training phase. During the project other associations and institutions could be involved in the project, with the aim of activating working groups, coordinating experiences and disseminating best practices.

4. Conclusions

In this work we presented OPS4Math, an Italian training project aimed at supporting the integration of the problem-solving approach in the mathematics teaching at Secondary Schools in Italy. The main aim of the project is to foster the "*teaching mathematics through problem solving*" strategy. Such aim will be pursued by what has been defined as the "*reversal of the didactical perspective",* i.e., starting from phenomena and problems to introduce curricular mathematical concepts in an appealing way. This is a very challenging task, since it requires a re-thinking of the classical way mathematics teaching is done in Italy (Akhter et al., 2015 and Foster, 2019). However, the authors are convinced that such approach is fundamental to foster STEM disciplines. Obviously, the authors are also aware that such "*reversal of the didactical perspective"* should be integrated and complemented

with other actions related to re-thinking also the way schoolbooks are written and the assessment system.

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